Nonequilibrium effective vector potential due to pseudospin exchange in graphene

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We show that exchange interactions in two-dimensional electron gases out of equilibrium can generate a fictitious vector potential with intriguing signatures in interference and Hall measurements. Detailed predictions are made for graphene, where the effect is enhanced by pseudospin exchange.

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By driving a system out of equilibrium one typically corrupts its quantum coherence. As a result many quantum effects observed in electronic structures, such as the Aharonov-Bohm interference and the Kondo effect are suppressed as one applies a voltage bias [1, 2, 3]. In rare instances, however, deviations from thermal equilibrium modify quantum effects in more interesting ways, or give rise to new coherent dynamics, such as the AC-Josephson effect [4]. In this Letter we predict a novel effect in the latter category: a fictitious vector potential induced by nonequilibrium conditions. The effect is particularly robust and intuitive in graphene [5, 6, 7], where the conduction electrons obey an equation that resembles the relativistic Dirac equation [8]. In that equation an orbital degree of freedom that is usually referred to as the 'pseudospin' plays the role of the conventional electron spin in the Dirac equation. This new quantum number has profound consequences, such as 'Klein tunneling', allowing electrons to pass arbitrarily high potential barriers [9], and a velocity renormalization through exchange interactions [10]. Here we show that in the nonequilibrium state created by a voltage bias the same pseudospin exchange interactions induce an effective vector potential. Unlike the fictitious vector potentials created by lattice defects or distortions in graphene [11, 12, 13] this effect does not cancel between the two bandstructure 'valleys', but it has directly observable consequences. We discuss two of those signatures: i) interference currents oscillating as a function of the current density in the material and ii) a 'Hall' voltage that reveals the fictitious magnetic field implied by the predicted vector potential in curved conductors.

The origin of this effect is best understood within the Dirac model for electrons in graphene at low energies,

$$H_\gamma = v\boldsymbol{\sigma_\gamma}\left(\boldsymbol{p} - \frac{e}{c}\boldsymbol{A}\right), \qquad (1)$$

where $\boldsymbol{\sigma_\gamma} = (\sigma_x, \gamma\sigma_y)$ is a vector of Pauli matrices in pseudospin space, $\boldsymbol{A}$ is the magnetic vector potential, $\gamma = \pm$ the valley index, $\boldsymbol{p}$ the electron momentum, and v, $-e$, c are the Fermi velocity, the electron charge, and the speed of light, respectively. Eq. (1) implies that in each valley electrons have a definite projection of their pseudospin onto their velocity. When a bias voltage is applied that drives an electrical current, such that electrons have a preferred direction of motion, a graphene sheet therefore carries an excess pseudospin. Consequently one expects a pseudospin-dependent exchange energy of electrons, described by an effective Hamiltonian with a vector potential $\boldsymbol{A}^{\mathbf{ex}}$ in addition to $\boldsymbol{A}$ in Eq. (1).

Exchange vector potential: To quantify the effect that is anticipated by the above simple Hartree-Fock argument we first consider a clean, infinite sheet of extrinsic graphene, with Fermi energy $\varepsilon_{\rm F} > 0$, at $\boldsymbol{A} = 0$. The effects of electron-electron interactions on single-particle quantities, as considered here, are captured by the electron self-energy Σ. We evaluate Σ in the so-called G_0W-approximation, resulting in a 'screened self-energy'. The 'Fock'-contribution to the retarded self-energy then is

$$\Sigma^{\rm R}_{\sigma\gamma}(\varepsilon,\boldsymbol{p}) = \frac{i}{2}\int\frac{d\omega}{2\pi}\frac{d^2\boldsymbol{k}}{(2\pi)^2}\Big[G^{(0)\rm K}_{\sigma\gamma}(\varepsilon-\omega,\boldsymbol{p}-\boldsymbol{k}) \times W^{\rm R}(\omega,\boldsymbol{k}) + G^{(0)\rm R}_{\sigma\gamma}(\varepsilon-\omega,\boldsymbol{p}-\boldsymbol{k})W^{\rm K}(\omega,\boldsymbol{k})\Big]. \qquad (2)$$

Here, the noninteracting retarded and Keldysh [14] Green functions $G^{(0)\rm R}$ and $G^{(0)\rm K}$, respectively, have an implicit matrix structure in pseudospin space and they are diagonal in the spin and valley indices σ and γ. The retarded and the Keldysh components of the RPA interaction potential $W^{\rm R} = v_{\rm C}/(1 - v_{\rm C}\Pi^{\rm R})$ and $W^{\rm K} = \Pi^{\rm K}|W^{\rm R}|^2$, respectively, are found from the unscreened Coulomb interaction $v_{\rm C}(\mathbf{q}) = 2\pi r_{\rm s} v/|\boldsymbol{q}|$ at interaction parameter $r_{\rm s}$ and the polarizability Π. The bare Keldysh Green function $G^{(0)\rm K}$ is characterized by occupation numbers $f_{s\boldsymbol{k}}$ of single-electron states $\psi_{\sigma\gamma s\boldsymbol{k}}$. Here, $\boldsymbol{k}$ are the electron momenta and $s = 1, -1$ for the electrons in the conduction band and the valence band, respectively. Through $G^{(0)\rm K}$ also $W^{\rm K}$ and $W^{\rm R}$ depend on $f_{s\boldsymbol{k}}$. We consider a state with current flow in the direction $\hat{\boldsymbol{w}}_{\boldsymbol{\theta}} = (\cos\theta, \sin\theta)$, $f_{s\boldsymbol{k}} = \Theta[\varepsilon_{\rm F} + \Theta(s\boldsymbol{k}\cdot\hat{\boldsymbol{w}}_{\boldsymbol{\theta}})eV - \varepsilon_{s\boldsymbol{k}}]$, with $\Theta(x) = 1$ for $x > 0$ and $\Theta(x) = 0$ for $x \le 0$. Here, $\varepsilon_{s\boldsymbol{k}}$ are the single-particle energies, V is the voltage that drives the current, and we take the limit of temperature $T = 0$, requiring $kT \ll eV$ in practice. We assume weak nonequilibrium, $eV \ll \varepsilon_{\rm F}$, such that we may expand in small deviations $\delta f_{s\boldsymbol{k}} = V(\partial f_{s\boldsymbol{k}}/\partial V|_{V=0})$ of the occupation numbers from their equilibrium values. We find the leading order (in $eV/\varepsilon_{\rm F}$) nonequilibrium contribution to the self-energy $\delta\Sigma^{\rm R}_{\sigma\gamma} = V(\partial\Sigma^{\rm R}_{\sigma\gamma}/\partial V|_{V=0})$ at small $r_{\rm s}$ by expanding the

$G^{(0)\mathrm{K}}$ of Eq. (2) in $\delta f_{s\boldsymbol{k}}$,

$$\delta\Sigma^{\mathrm{R}(0)}_{\sigma\gamma}(\varepsilon,\boldsymbol{p}) \;=\; -\frac{1}{2}\int\frac{d^2\boldsymbol{k}}{(2\pi)^2}\sum_s \delta f_{s,\boldsymbol{k}}\begin{pmatrix}1 & se^{i\gamma\phi}\\ se^{-i\gamma\phi} & 1\end{pmatrix} \times W^{\mathrm{R}}(\varepsilon-\varepsilon_{s\boldsymbol{k}},\boldsymbol{p}-\boldsymbol{k}). \quad (3)$$

Here, $\boldsymbol{k} = k\hat{\boldsymbol{w}}_{\boldsymbol{\phi}}$ and W^{R} takes its equilibrium form, obtained in Refs. [15, 16]. At $eV \ll \varepsilon_{\mathrm{F}}$, $\delta f_{s\boldsymbol{k}}$ is nonvanishing only if $|\varepsilon_{s\boldsymbol{k}} - \varepsilon_{\mathrm{F}}| \ll \varepsilon_{\mathrm{F}}$. For all quantities computed below Σ is needed, in the same limit, at $|\varepsilon - \varepsilon_{\mathrm{F}}| \ll \varepsilon_{\mathrm{F}}$ and thus $|\varepsilon - \varepsilon_{s\boldsymbol{k}}| \ll \varepsilon_{\mathrm{F}}$ in Eq. (3). Since the typical momentum transfer $\boldsymbol{p}-\boldsymbol{k}$ in Eq. (3) is of order k_{F}, we may substitute $W^{\mathrm{R}}(\varepsilon-\varepsilon_{s\boldsymbol{k}},\boldsymbol{p}-\boldsymbol{k}) \approx W^{\mathrm{R}}(0,\boldsymbol{p}-\boldsymbol{k}) = 2\pi r_{\mathrm{s}} v/(|\boldsymbol{p}-\boldsymbol{k}|+q_{\mathrm{sc}})$ into Eq. (3), with the screening wavevector $q_{\mathrm{sc}} = 4r_{\mathrm{s}}k_{\mathrm{F}}$ at Fermi wavevector k_{F} [15, 16]. The pseudospin-diagonal components of the real part $\mathrm{Re}\,\delta\Sigma^{\mathrm{R}(0)}$ shift the chemical potential. The off-diagonal components of the hermitian part of $\delta\Sigma^{\mathrm{R}(0)}$ have a p-wave contribution that implies the velocity renormalization analyzed in Ref. [10] and an s-wave part that acts as an effective vector potential $\boldsymbol{A}^{\mathbf{ex}}$,

$$\boldsymbol{A}^{\mathbf{ex}} = -\frac{c}{2ev}\mathrm{Re}\int_0^{2\pi}\frac{d\vartheta}{2\pi}\,\mathrm{Tr}\,\boldsymbol{\sigma}_{\boldsymbol{\gamma}}\delta\Sigma^{\mathrm{R}(0)}_{\sigma\gamma}(k_{\mathrm{F}}\hat{\boldsymbol{w}}_{\boldsymbol{\vartheta}},\varepsilon_{\mathrm{F}}) \quad (4)$$

(we set $\hbar = 1$), which, for the above $f_{s\boldsymbol{k}}$, evaluates to

$$\boldsymbol{A}^{\mathbf{ex}} = \zeta\frac{c}{v}\frac{V}{8\pi}\hat{\boldsymbol{w}}_{\boldsymbol{\theta}},\;\; \zeta = \int_0^{2\pi}\frac{d\vartheta}{2\pi}\,\frac{4r_{\mathrm{s}}}{\sqrt{2-2\cos\vartheta}+4r_{\mathrm{s}}}. \quad (5)$$

At $r_{\mathrm{s}} \ll 1$ we have $\zeta \sim 4r_{\mathrm{s}}\ln(\pi/8r_{\mathrm{s}})/\pi$ and in the range of $0.15 < r_{\mathrm{s}} < 2.4$ typical for graphene [17] ζ is of order unity ($\zeta \approx 0.38$ at $r_{\mathrm{s}} = 0.15$ and $\zeta \approx 0.89$ at $r_{\mathrm{s}} = 2.4$). Note that the valley-dependence of the effect indeed drops out, as advertised [18].

Expanding the W^{R} and the W^{K} of Eq. (2) in $\delta f_{s\boldsymbol{k}}$ results in corrections to ζ. These corrections take the form of integrals over higher powers of W^R than the leading contribution and they are thus negligible at $r_{\mathrm{s}} \ll 1$, when the G_0W-approximation is reliable [19]. We therefore employ our 'screened Hartree-Fock' approximation Eq. (3) in the remainder of this Letter. The Keldysh self-energy Σ^{K} and the anti-hermitian part of Σ^{R} account for inelastic relaxation. These inelastic processes are negligible in the geometries we study below: graphene ribbons with a finite length L that are ballistic, that is they have long inelastic and elastic mean free paths l_{in} and l_{el}, respectively, $l_{\mathrm{in}}, l_{\mathrm{el}} \gg L$, and they are well-coupled to electron reservoirs [20]. Neglecting relaxation by phonons ($T = 0$), we find from Eq. (2) for a transport electron with momentum $\boldsymbol{k} = k\hat{\boldsymbol{w}}_{\boldsymbol{\phi}}$ in the above nonequilibrium state $f_{s\boldsymbol{k}}$ at $r_{\mathrm{s}} \ll 1$ and $eV \ll r_{\mathrm{s}}\varepsilon_{\mathrm{F}}$,

$$l_{\mathrm{in}}^{-1} = k_{\mathrm{F}}\frac{\delta\varepsilon^2(3eV-2\delta\varepsilon)}{3\pi\varepsilon_{\mathrm{F}}^3}\sum_{\eta=\pm1}\left(\frac{r_{\mathrm{s}}}{|\theta-\phi+\eta\pi/2|+4r_{\mathrm{s}}}\right)^2 \quad (6)$$

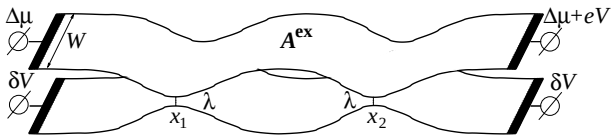

FIG. 1: Two ballistic graphene ribbons attached to large leads (black bars), biased by a voltage V such that a current flows through the upper ribbon. Electrons tunnel between the ribbons at x_1 and x_2 with matrix element λ, driven by a voltage δV. The amplitudes for electron propagation between x_1 and x_2 along the upper ribbon with exchange vector potential $\boldsymbol{A}^{\mathbf{ex}}$ interfere with those along the lower ribbon, causing oscillations of the tunnel current as a function of V.

with $\delta\varepsilon = \varepsilon_{s\boldsymbol{k}} - \varepsilon_{\mathrm{F}}$, valid at $|\theta - \phi \pm \pi/2| \gg eV/\varepsilon_{\mathrm{F}}$ [20].

An effective vector potential $\boldsymbol{A}^{\mathbf{ex}}$ is generally expected in electron systems where the exchange energy between two electrons depends on their relative motion. This requires $q_{\mathrm{sc}} \lesssim k_{\mathrm{F}}$, which is typically fulfilled in n-type GaAs, but not in p-type GaAs or Si-MOSFETs [21]. In graphene the effect persists into the strong screening limit $q_{\mathrm{sc}} \gg k_{\mathrm{F}}$ (at $r_{\mathrm{s}} \gg 1$) through pseudospin exchange.

Interference: We now turn to a discussion of signatures of the predicted effective vector potential. We first consider two tunnel-coupled, ballistic graphene ribbons, as shown in Fig. 1, where $\boldsymbol{A}^{\mathbf{ex}}$ can be observed interferometrically [22]. The two ribbons are well-coupled to electron reservoirs at their ends and they touch at x_1 and x_2, which creates a weak tunnel coupling λ between them. For simplicity we assume λ to be identical at x_1 and x_2 and constant over the width W of the ribbons. We take the semiclassical limit $k_{\mathrm{F}}W \gg 1$ and allow for different path lengths Δx^{u} and Δx^{l} between the points of tunneling along the upper and the lower ribbon, respectively. A voltage $V \ll \varepsilon_{\mathrm{F}}/e$ that leaves the mean electric potential invariant ($\Delta\mu = -eV/2 + \sqrt{\varepsilon_{\mathrm{F}}^2 - (eV/2)^2} - \varepsilon_{\mathrm{F}}$ in Fig. 1) drives a current I through the upper ribbon. According to Eq. (5) an $\boldsymbol{A}^{\mathbf{ex}}$ results that shifts the phase of an electron traveling from x_1 to x_2 along the upper ribbon by $\Phi = \zeta eV\Delta x^{\mathrm{u}}/8\pi v$. The tunneling current I_{tun} between the ribbons, driven by a small voltage δV, is expected to oscillate as a function of Φ. We accordingly find

$$G_{\mathrm{tun}} \propto |\lambda|^2 k_{\mathrm{F}} W e^2 \int_{1/N}^{1-1/N}\frac{dq}{1-q^2}\left\{1+q^2+\cos\Phi\cos\left[k_{\mathrm{F}}(\Delta x^{\mathrm{u}}-\Delta x^{\mathrm{l}})\sqrt{1-q^2}\right]\right\} \quad (7)$$

for the differential tunnel conductance $G_{\rm tun} = dI_{\rm tun}/d\delta V|_{\delta V=0}$ at generic $\varepsilon_{\rm F}$ and $|\Delta x^{\rm u} - \Delta x^{\rm l}| \ll \Delta x^{\rm u}$ [20]. Here $1/N \sim 1/k_{\rm F}W$ depends on the Fermi wavevector of the lower ribbon $k_{\rm F}$. We conclude that in the geometry of Fig. 1 the predicted exchange vector potential $\boldsymbol{A}^{\rm ex}$ has the very distinctive signature that the tunnel conductance $G_{\rm tun}$ *between* the two ribbons oscillates as a function of the current flowing *through* the upper ribbon.

The visibility of the predicted oscillations is quantified by the interference contrast $C = G_{\rm int}/G_{\rm dir}$. We extract the 'direct' conductance $G_{\rm dir}$ and the oscillatory 'interference' conductance $G_{\rm int}$ from Eq. (7) as the first two and the third terms in curly brackets, respectively. We find $C \sim 1$ for $k_{\rm F}\Delta x^{\mu} \gg \sqrt{N}$ ($\mu \in \{{\rm u},{\rm l}\}$), but $k_{\rm F}|\Delta x^{\rm u} - \Delta x^{\rm l}| \ll \sqrt{N}$. Larger path length differences $k_{\rm F}\Delta x^{\mu} \gg k_{\rm F}|\Delta x^{\rm u} - \Delta x^{\rm l}| \gg \sqrt{N}$ suppress the interference contrast, $C \sim \sqrt{N}/k_{\rm F}|\Delta x^{\rm u} - \Delta x^{\rm l}| \ln N$. Above we assumed tunneling matrix elements λ that are constant over the width of the ribbons, so that electrons tunnel only between identical transverse modes. To estimate the influence of inhomogeneities of λ we compute the tunneling conductance also for random tunnel matrix elements with identical ensemble averages for scattering between any two of the transverse modes. We find $C \sim 1/N$ and we expect that C suffers similar or weaker suppressions for most realistic forms of tunnel contacts.

Violations of our limit $\Delta x^{\mu} \lesssim l_{\rm in}$ suppress C exponentially. According to Eq. (6) the condition $\Delta x^{\mu} \lesssim l_{\rm in}$ is met at low voltages. Demanding at the same time that V be large enough to produce one entire oscillation of $G_{\rm tun}$ imposes a lower bound on the path lengths $k_{\rm F}\Delta x^{\mu} \gtrsim 32\pi^2 r_{\rm s}^{1/2}\zeta^{-3/2}$ [23]. Note that Eq. (6) assumes a two-dimensional electron gas, $W \gg v/eV$. In the one-dimensional limit, realizable in carbon nanotubes, quasiparticles with well-defined energies cease to exist and the above predictions do not apply. Inelastic or elastic scattering that induces a potential drop ΔV along the ribbons may also cause an electric Aharonov-Bohm effect. In order for the resulting oscillations not to obscure the predicted effect it has to be assured that $\Delta V \ll v/e\Delta x^{\mu}$.

Hall effect: In a curved conductor, cut out of a sheet of graphene, the current direction $\boldsymbol{w}_{\boldsymbol{\theta}}$ is not constant anymore. We thus expect an exchange vector potential $\boldsymbol{A}^{\rm ex}$ similar to Eq. (5), but space-dependent. The effective magnetic field that would generically result should be observable by the Hall effect that it causes. To confirm this scenario we consider a segment of a graphene annulus with inner radius R, radial width W, and angular width β (see Fig. 2). We again address the semiclassical limit $k_{\rm F}W \gg 1$ [22] and omit the details of the calculation that will be presented elsewhere [20]. The segment is well-coupled to electron reservoirs at its angular boundaries, such that electrical current flows in the angular direction and we have periodic boundary conditions at the angular edges. The wavefunctions with these boundary conditions obtain from the solutions of Eq. (1) in

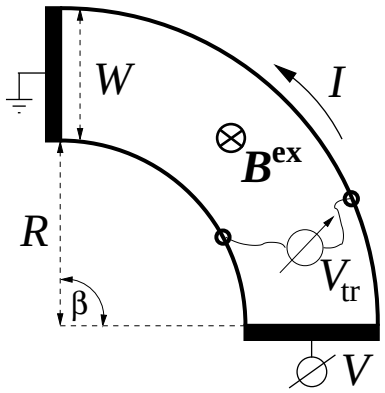

FIG. 2: Graphene annulus of width W and inner radius R, contacted by electron reservoirs (solid black bars) whose electrochemical potentials differ by eV. A current I is flowing that produces a transverse voltage $V_{\rm tr}$ by a Hall effect in the (fictitious) exchange magnetic field $\mathbf{B}^{\rm ex}$.

the full annulus, where angular momentum l is a good quantum number. Additionally the single-particle states $\psi_{\sigma\gamma snl}$ are characterized by a radial quantum number n and they have eigenenergies ε_{snl}. At $r_{\rm s} = 0$ a voltage bias between the two reservoirs in Fig. 2 creates an occupation of states $f_{snl} = \Theta[\varepsilon_{\rm F} + \Theta(sl)eV - \varepsilon_{snl}]$ ($T = 0$). Substituting the corresponding $G^{(0)K}$ into Eq. (2), in our 'screened Hartree-Fock' approximation corresponding to Eq. (3), and extracting $\boldsymbol{A}^{\rm ex}$ through Eq. (4) yields an exchange vector potential $\boldsymbol{A}^{\rm ex}(r\hat{\boldsymbol{w}}_{\boldsymbol{\varphi}}) = \zeta(c/v)(V/8\pi)\hat{\boldsymbol{w}}_{\boldsymbol{\varphi+\pi/2}}$ as suggested by Eq. (5), pointing in angular direction.

The above $\boldsymbol{A}^{\rm ex}$ implies an effective magnetic field $B^{\rm ex} = |\boldsymbol{A}^{\rm ex}|/r$ perpendicular to the annulus. The corresponding Lorentz force modifies the charge density ρ, found from the Green functions implied by $\Sigma^{(0)}$, Eq. (3),

$$\rho(r) = \sum_{\sigma\gamma snl} f^{\rm ex}_{snl} |\psi^{\rm ex}_{\sigma\gamma snl}(r)|^2. \tag{8}$$

Here, $\psi^{\rm ex}_{\sigma\gamma snl}$ and $f^{\rm ex}_{snl}$ follow from Eq. (1) with $\boldsymbol{A} = \boldsymbol{A}^{\rm ex}$ and in the semiclassical limit we obtain

$$\rho(r) = \rho_0 - \frac{1}{2\pi}\left(\frac{eV}{2\pi v}\right)^2 \left[\frac{R}{r}\frac{R+W}{W}\ln\left(1+\frac{W}{R}\right) - 1\right]. \tag{9}$$

The equilibrium contribution ρ_0 is r-independent [24]. The r-dependent contribution to ρ, Eq. (9), is screened by a charge rearrangement through a transverse voltage

$$V^{\rm ex}_{\rm tr} = \frac{V}{8\pi^2}\frac{eV}{\varepsilon_{\rm F}}\ln\left(1+\frac{W}{R}\right) + \mathcal{O}(e^3V^3/\varepsilon_{\rm F}^2), \tag{10}$$

restoring charge neutrality. Eq. (10) predicts a transverse 'Hall' voltage generated by an electrical current in the absence of a real magnetic field. It is a second peculiar signature of the exchange vector potential $\boldsymbol{A}^{\rm ex}$.

Eq. (10) crucially relies on our assumption that the annulus is ballistic, $l_{\rm m} \gg L$, where $L = \max\{\beta R, W\}$

and $l_{\rm m} = \min\{l_{\rm el}, l_{\rm in}\}$.In the opposite limit of a diffusive annulus, $L \gg l_{\rm m}$, the current density decays as $1/r$, implying $\boldsymbol{A}^{\rm ex} \propto \hat{\boldsymbol{w}}_{\boldsymbol{\varphi}+\boldsymbol{\pi}/\boldsymbol{2}}/r$ with $B^{\rm ex} = 0$ and thus $V_{\rm tr}^{\rm ex} = 0$. With moderately strong backscattering, $l_{\rm m} \sim L$, $\boldsymbol{A}^{\rm ex}$ still produces a transverse voltage. Backscattering, however, induces a voltage drop along the annulus that generically produces an additional transverse voltage $V_{\rm tr}^{\rm scatt}$. The linear in V contribution to $V_{\rm tr}^{\rm scatt}$ can be eliminated by measuring $\bar{V}_{\rm tr} = [V_{\rm tr}(V) + V_{\rm tr}(-V)]$. Any energy-dependence of the scattering, however, will add a quadratic in V contribution to $V_{\rm tr}^{\rm scatt}$ that competes with $V_{\rm tr}^{\rm ex}$. Typically backscattering in graphene has an energy-dependence on the scale $\varepsilon_{\rm F}$ [25, 26, 27, 28]. It follows that $\bar{V}_{\rm tr}^{\rm scatt}$ is negligible, $\bar{V}_{\rm tr}^{\rm scatt} \ll \bar{V}_{\rm tr}^{\rm ex}$, if $L/l_{\rm m} \ll \ln(1 + W/R)$. Experimentally $\bar{V}_{\rm tr}^{\rm scatt}$ can be distinguished from $V_{\rm tr}^{\rm ex}$ by varying $\varepsilon_{\rm F}$ at $eV \ll \varepsilon_{\rm F}$. If the measured $\bar{V}_{\rm tr}$ is due to energy-dependent scattering, a variation of $\varepsilon_{\rm F}$ by $d\mu \simeq eV$ causes a relative change of $\bar{V}_{\rm tr}$ by $\Delta\bar{V}_{\rm tr}^{\rm scatt}/\bar{V}_{\rm tr}^{\rm scatt} = (\bar{V}_{\rm tr}^{\rm scatt}(V)|_{\varepsilon_{\rm F}+d\mu} - \bar{V}_{\rm tr}^{\rm scatt}(V)|_{\varepsilon_{\rm F}})/\bar{V}_{\rm tr}^{\rm scatt}(V)|_{\varepsilon_{\rm F}} \simeq 1$. In contrast, the relative change is only $\Delta\bar{V}_{\rm tr}^{\rm ex}/\bar{V}_{\rm tr}^{\rm ex} \simeq d\mu/\varepsilon_{\rm F} \ll 1$ if $V_{\rm tr}$ is due to the exchange interaction.

A second effect competing with $V_{\rm tr}^{\rm ex}$ is the Hall effect in the (real) magnetic field produced by the current flowing in the annulus. In the limit $W/R \ll 1$ the relative magnitude of the resulting transverse voltage $\bar{V}_{\rm tr}^{\rm magn}$ is

$$\frac{\bar{V}_{\rm tr}^{\rm magn}}{\bar{V}_{\rm tr}^{\rm ex}} = \frac{8}{\zeta\pi}\left(\frac{v}{c}\right)^2 \frac{e^2}{v}\left[\ln\frac{8R\tan(\beta/4)}{W}) + \frac{3}{2}\right] k_{\rm F} W \tag{11}$$

up to corrections of order $(W/R)\ln(W/R)$. For the parameters $W = 0.2R$, $\beta = \pi/6$, and $r_{\rm s} = 0.15$ we find for instance $\bar{V}_{\rm tr}^{\rm magn}/\bar{V}_{\rm tr}^{\rm ex} \approx 4 \cdot 10^{-4}\, k_{\rm F} W$. For typical sample dimensions of $W \approx 1\mu$m this effect is thus negligible up to the largest $k_{\rm F} \approx 1\,$nm. Also the smallness of $\bar{V}_{\rm tr}^{\rm magn}$ can be verified experimentally by its distinctive dependence on $\varepsilon_{\rm F}$. In this case $\bar{V}_{\rm tr}^{\rm magn}$ is independent of $\varepsilon_{\rm F}$, $\Delta\bar{V}_{\rm tr}^{\rm magn}/\bar{V}_{\rm tr}^{\rm magn} = 0$, compared to $\Delta\bar{V}_{\rm tr}^{\rm ex}/\bar{V}_{\rm tr}^{\rm ex} \simeq d\mu/\varepsilon_{\rm F}$. Both competing effects discussed above can be ruled out by simultaneously varying $\varepsilon_{\rm F}$ and V.

Conclusions: We have predicted a novel nonequilibrium effect in coherent two-dimensional electronic systems. The effect is due to electron-electron interactions and it generates a fictitious vector potential that has striking signatures: interference currents that oscillate as a function of the current density in the material and a 'Hall' voltage in zero magnetic field. We have made detailed predictions for graphene, where the effect is particularly robust owing to the pseudospin degree of freedom of the conduction electrons.

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